Available online at www.sciencedirect.com

SciVerse ScienceDirect

Energy Procedia 18 (2012) 225 – 234

Energy Procedia

Deterministic optimal management strategy of hydroelectric power plant

A. Bensalem[a,*], A. Bouhentala[a], A. El-Maouhab[b]

[a] Electrical engineering department. University of Hadj Lakhdar. 05000 Batna, Algeria
[b] Department of Electrical Engineering.University of houari Boumèdienne, 16000 Algiers, Algeria.

Abstract

In this paper, we make a comparison between two management strategies of hydroelectric power plan consisting of multi-reservoirs. The first strategy is base on the maximization of the reservoir contents at the end of the exploitation horizon. The second strategy is base on the maximization of the reservoir contents at the end of the sub-periods of the same exploitation horizon.

To solve such problem, we propose a new objective function model which permits to minimize the use of water. This model is based on the enhancement of the water value in function of its location in any reservoirs of the system and of the waterfall height. Hence, the objective function is represented in function of the water potential energy stored in all reservoirs.

Keywords: Optimal management strategy, potential energy, exploitation horizon;

1. Introduction

The short-term optimal operating strategy of hydroelectric power systems is a determinist problem [1-2], which consists of choosing the preliminary selected quantity of water to release from each reservoir of the system over the planning horizon, in order to meet an hourly electric power demand assigned previously. The prime objective here is to perform the operating strategy with the lowest use of water, which is achieved by avoiding spilling and by maximizing the hydropower generation. In addition, all

 doi:10.1016/j.egypro.2012.05.034

operating constraints must be satisfied. The maximization of hydropower generation is achieved by maximizing the heads. Consequently, this allows maximizing the reservoirs content.

To reach this objective, two management strategies are proposed. The first strategy is base on the maximization of the reservoir contents at the end of the planning horizon. The second strategy is base on the maximization of the reservoir contents at the end of the sub-periods of the same planning horizon.

In order, to improve the performances of the management strategy, it is suggested to subdivide the short-term planning horizon into shorter horizons or sub-periods. Thus, the objective becomes the maximization of the potential energy value stored at the end of the sub-periods horizons. Then, the final state of the sub-period horizon will be regarded as the initial state of the next sub-period horizon and so on. Hence, a reduced size problem is solved in each sub-period horizon. The scheduling results of the two strategies are presented and are compared.

When modeling the problem, and for more accuracy, the following factors, which make the problem more complex, are taken into consideration; significant water travel time between reservoirs, the multiplicity of the input-output curve of hydroelectric reservoirs that have variable heads, the maximum generation of the hydropower plant varies with the hydraulic head i.e. the quantity of water required for a given power output decreases as the hydraulic head increases, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir, whether the reservoirs have very different storage capacity and whether the system has quite complex topology with many cascaded reservoirs.

To solve the short-term operating strategy problem, the discrete maximum principle [3-4] is used. While solving the equations relating to the discrete maximum principle, the gradient method [3] has been used. However, to treat equalities constraints the Lagrange's multiplier method is used. And the inequalities constraints are treated by using the augmented Lagrangian method [5-8].

The hydroelectric power system considered in this paper consists of ten reservoirs hydraulically coupled, i.e., the release of an upstream reservoir contributes to the inflow of downstream reservoirs. All reservoirs are located in the same river. The time taken by water to travel from one reservoir to the downstream reservoir [9] and the water head variation are taken into account. The natural inflow and the demand for electrical energy are known beforehand. The scheduling is stretched over one week divided into hours.

The decision variables in the optimization problem are the amount of water to be released from each reservoir to their direct downstream reservoirs in a given period. The state variables are the contents of the reservoirs.

Nomenclature

$E_p(x_i^{k_f}, h_i^{k_f})$ potential energy of water stored in reservoir i at the end of the planning horizon k_f. This energy depends on the amount of water stored in the reservoir i, on its effective water head $h_i^{k_f}$ and on the effective water head of the downstream reservoirs.

$x_i^{k_f}$	content of the reservoir i at the end of period k_f, in Mm^3.
n	number of reservoirs of the system.
k_f	the last hour of the planning horizon, in hours.
m	the reservoir immediately preceding the reservoir i.

u_{mi}^{k}, v_{mi}^{k} respectively the discharge and the spilled outflows from the upstream reservoir *m* incoming later to the downstream reservoir i during period k, in Mm^3.

S_{mi} time required for the water discharged from reservoir *m to* reach its direct downstream reservoir i, in hours.

$\sum E_p^k(u_{mi}^k, v_{mi}^k)$ total potential energy of the outflow from reservoir *m*, which will reach later the downstream reservoir i after the last hour of the planning horizon k_f.

u_i^k discharge from reservoir i during period k, in Mm^3.

v_i^k spillage from reservoir i during period k, in Mm^3.

q_i^k total inflow to the reservoir i during period k, in Mm^3.

e the extreme upstream reservoirs.

y_i^k Natural inflow to the reservoir i during period k, in Mm^3.

$u_{mi}^{k-s_{mi}}, v_{mi}^{k-s_{mi}}$ The discharge and the spilled outflows, respectively, from the upstream reservoir m incoming later to the downstream reservoir i during period k, in Mm^3.

$\underline{x}_i, \overline{x}_i$ lower and upper bounds on reservoir storage capacity, respectively, for reservoir i, in Mm^3.

$\underline{u}_i, \overline{u}_i$ minimum and maximum bound on water discharge, respectively, of hydro power plant i, in Mm^3.

x_i^0 initial content of reservoir i.

D^k system load demand at each period k, in Mw.

P_i^k Electric power generated by hydro plant i at period k, in Mw. The generation is a function of the water discharge u_i^k and of the effective water head u_i^k.

r penalty weight.

$\rho_i^{k\,(j+1)}$ Lagrange multipliers at iteration $j+1$.

α pre-selected step size.

$h_i^k(x_i^k)$ Effective water head of hydropower plant i at period k.

2. Mathematical model formulation

The main objective of the short-term operating strategy of hydroelectric power system is to maximize the reservoir's contents which imply maximizing the value of potential energy stored at the end of the planning horizon, while satisfying demand for electrical energy and all other specified constraints. Thus, the suggested mathematical model for the deterministic short-term operating strategy of the hydroelectric power systems is as follows:

2.1. The objective function

The main objective is to maximize the total potential energy of water stored in all the reservoirs. The formulation must take into account the fact that the water stored in one reservoir will be re-used in all its downstream reservoirs, hence, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir, hence:

$$\max \sum_{i=1}^{n} E_P(x_i^{k_f}, h_i^{k_f}) + \sum_{k=k_f-S_{mi}}^{k_f} E_p(u_{mi}^k, v_{mi}^k)$$

2.2. Operational constraints [1-2][10-15]:

- Hydraulic continuity constraint:

The flow balance equation of each reservoir i of the system, for every period k is represented by the following hydraulic continuity equation:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k - v_i^k$$

Total inflow to the reservoir i during period k, is :

$$q_i^k = \begin{cases} y_i^k & \text{if } i \le e. \\ y_i^k + \sum_m (u_{mi}^{k-s_{mi}} + v_{mi}^{k-s_{mi}}) & \text{otherwise.} \end{cases}$$

- Limits on storage capacity of each reservoir i :

$$\underline{x}_i \le x_i^k \le \overline{x}_i$$

- Limits on discharged outflow of hydro plant i :

$$\underline{u}_i \le u_i^k \le \overline{u}_i$$

- Load constraints:

The total power generated by all the hydroelectric plants must satisfy the system load demand at each period of the planning horizon. In mathematical terms, this has the following form:

$$\sum_{i=1}^{n} P_i^k = D^k$$

2.3. Modeling the short-term operating strategy problem

The suitable mathematical model proposed for the short-term scheduling problem of a hydroelectric power system is as follows:

$$\max \sum_{i=1}^{n} E_p(x_i^{k_f}, h_i^{k_f}) + \sum_{k=k_f-S_{mi}}^{k_f} E_p(u_{mi}^k) \quad (1)$$

Subject to the following constraints:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k \tag{2}$$

$$\sum_{i=1}^{n} P_i^k = D^k \tag{3}$$

$$0 \le u_i^k \le \overline{u}_i \tag{4}$$

$$\underline{x}_i \le x_i^k \le \overline{x}_i \tag{5}$$

To avoid the spillage, we make v_i^k equal to zero.

3. Solution method

The problem (1)-(3) is solved by using the discrete maximum principle as follows [3-8]:
The constraint (2) is associated to the criterion (1) with the dual variable λ_i^k. Furthermore, to satisfy the balance between electric power demand and generation, the constraint (3) is associated to the criterion (1) with the Lagrange multiplier β^k, and then the function H^k called the Hamiltonian function is defined and has the following form:

$$H^k = \sum_{i=1}^{n} [\lambda_i^k (x_i^k + q_i^k - u_i^k] + \beta^k (\sum P_i^k - D^k) \tag{6}$$

Where u_i^k and x_i^k represent respectively the control and state variables.
To take into account the possible violation of constraint (5) the following procedure is used:
The two-sided inequality constraint (5) can be broken into two inequalities constraints and rewritten, following the substitution of equation (2) for x_i^k:

$$(x_i^{k-1} + q_i^k - u_i^k) - \overline{x}_i \le 0 \tag{7}$$

$$(x_i^{k-1} + q_i^k - u_i^k) - \underline{x}_i \ge 0 \tag{8}$$

To treat these inequalities constraints the Augmented Lagrangian method is used [5-7]. It consists on adding the functions R_i^k and Q_i^k to the Hamiltonian H^k that penalizes respectively the violations of the inequalities constraints (7) and (8), i.e., the violation of lower and upper limits of the original constraint (5). Then the Hamiltonian H^k becomes as follows:

$$H^k = \sum_{i=1}^{n} [\lambda_i^k (x_i^k + y_i - u_i^k)] + \beta^k (\sum P_i^k - D^k) + R_i^k + Q_i^k \tag{9}$$

The penalty function R_i^k is defined as follows [5-7]:

$$R_i^k = \rho_i^k \Psi_i^k + r(\Psi_i^k)^2 \tag{10}$$

The Lagrange multipliers at iteration j+1 is updated as follows:

$$\rho_i^{k\,(j+1)} = \rho_i^{k\,(j)} + 2r \max(x_i^k - \overline{x}_i, -\frac{\rho_i^{k\,(j)}}{2r}) \tag{11}$$

The function Ψ_i^k is determined as follows:

$$\Psi_i^k = \max(x_i^k - \overline{x}_i, -\frac{\rho_i^k}{2r}) \tag{12}$$

The penalty function Q_i^k is calculated in the same manner as R_i^k.

The problem (1)-(5) becomes:

$$\max H^k \tag{13}$$

The necessary conditions for the optimum are:

$$\frac{\partial H^k}{\partial u_i^k} = 0 \tag{14}$$

To find the optimal water discharge trajectory u_i^k from Eq. (14), we must solve the difference's equations (2) and the following equation called the adjoint equation [3] must be solved together:

$$\lambda_i^{k-1} = \frac{\partial H^k}{\partial x_i^{k-1}} \qquad (15)$$

The boundary conditions for equation (2) and (15) are:

- The first one is the initial state, which is specified, i.e., the initial content of all reservoirs is known, thus:

$$x_i^k = a_i \qquad (16)$$

- The second one is the terminal condition for the adjoint equation:

$$\lambda_i^{k_f} = \frac{\partial E_p(x_i^{k_f})}{\partial x_i^{k_f}} \qquad (17)$$

The necessary conditions for the optimality constitute a two-point boundary value problem, where its solution determines the optimal state and control variables. This problem is solved iteratively by using the gradient method [3].

The proposed algorithm procedure for the second strategy proceeds as follow:

Step 1 Initialization of the first sub-period, j=1.

Step 2 Fixation of the parameters α , r and initialization of the multipliers β^k and ρ_i^k , for $i = 1, ..., n$ and $k = 1, \dots, k_f$.

Step 3 Selection of an admissible control trajectory u_i^k for $i = 1, ..., n$ and $k = 1, ..., k_f$.

Step 4 Utilization of the known initial contents x_i^0 and the selected control trajectory u_i^k , solve Eq. (2) forward in time to obtain x_i^k .

Step 5 Utilization of the known u_i^k, x_i^k and the terminal condition (10), solve equation (8) backward in time to obtain λ_i^k .

Step 6 Adjustment of the multiplier β^k so as to have equilibrium between the demand and the generation.

Step 7 Utilization of the known u_i^k , x_i^k , λ_i^k and the adjusted β^k , to compute for all i and k the gradient G_i^k from Eq. (14).

Step 8 Computation of the new trajectory $\tilde{u}_i^k$ using the following expression:

$$\tilde{u}_i^k = u_i^k + \alpha \cdot G_i^k \qquad (18)$$

Step 9 If some values of the new control variable $\hat{u}_i^k$ that satisfy the optimality condition (14) violate the inequality constraint (4), we fix their values to their limits. The others are left free. Then we re-adjust the value of the multiplier β^k in order to satisfy constraint (3).

Step10 Computation for all i and k the new value of the gradient G_i^k as done in step 6 using the new value of the multiplier β^k calculated in step 9.

Step11 Computation of the new trajectory $\hat{u}_i^k$ using the following expression:

$$\hat{u}_i^k = Max\left[0, \min\left\{\bar{u}_i, u_i^k + \alpha \cdot G_i^k\right\}\right]$$

If $Max\left[\hat{u}_i^k, u_i^k\right]$ is greater than the desired accuracy limit, then set $u_i^k = \hat{u}_i^k$ and go back to step 3.

Step12 Verification if the state constraints (5) are satisfied within the desired accuracy limits. If not, update the value of the Lagrange multipliers ρ_i^k and go back to step 3.

Step13 Verify whether the algorithm is performed for all the periods. If they are not, update the period ($j = j+1$) and go back to step 4.

Step14 Printing the results.

The algorithm proposed for the first strategy is the same as the one of the second strategy but without the steps 1 and 13.

It is specified that the algorithm adjusts automatically the weights during the search progress starting from an initial weight.

4. Test results

The both described algorithms of the strategies are implemented in FORTRAN. In order to compare their efficiency, we apply both of them to the same system, which is composed of ten reservoirs located on the same river as shown in Fig. 1.

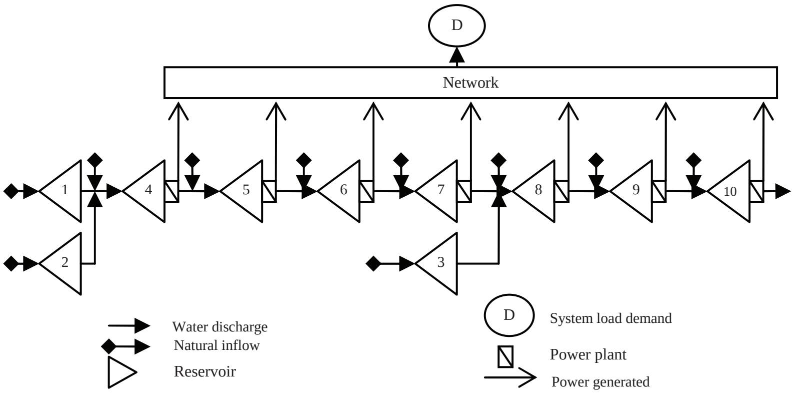


Fig.1: The reservoir network.

The characteristics of the reservoirs and water time travel are shown in table 1. The natural inflows are assumed constant throughout the week in all reservoirs. Their values are depicted in table 2 as well as the initial contents of each reservoir.

The electrical power produced in MW at the hydroelectric plant i during a period k is given by the following expression:

$$P_i(h_i^k, u_i^k) = h_i^k(x_i^k) \cdot u_i^k \tag{19}$$

The hourly demand D^k in MW is shown in Fig. 2.

Table 1. Characteristics of the installations.

i	$\bar{x}_i$ (M.m³)	$\bar{u}_i$ (M.m³/h)	h_i (m)	S_{mi} (h)
1	8777,2	1,1232	0,00	55
2	986,4	0,5272	0,00	70
3	998,0	0,5054	0,00	42
4	504,9	2,5531	66,61	5
5	8,5	2,4181	114,18	7
6	4,2	2,5650	92,41	2
7	4,8	2,5240	83,28	22
8	26,9	2,7648	55,72	3
9	4,54	3,0476	107,66	2
10	3,4	3,4686	40,81	0

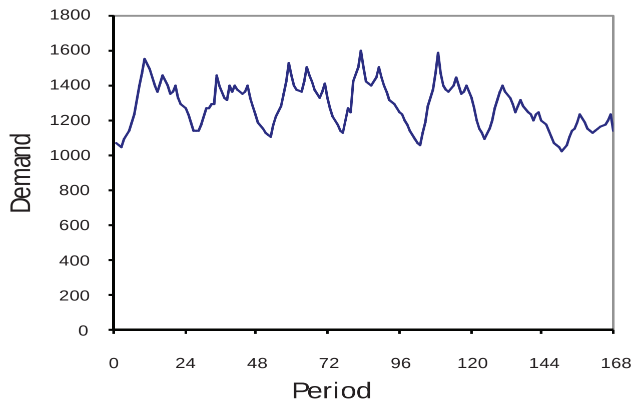


Fig. 2: Hourly demand profile during one week.

5. Implementation of results

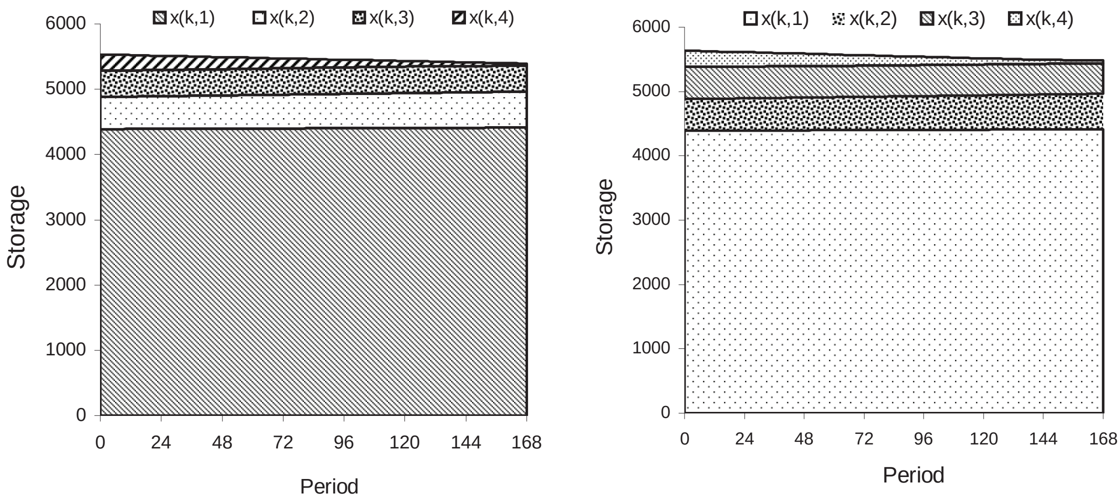
In this section, we present the results obtained from the implementation of the proposed strategy's algorithms described in the preceding section.

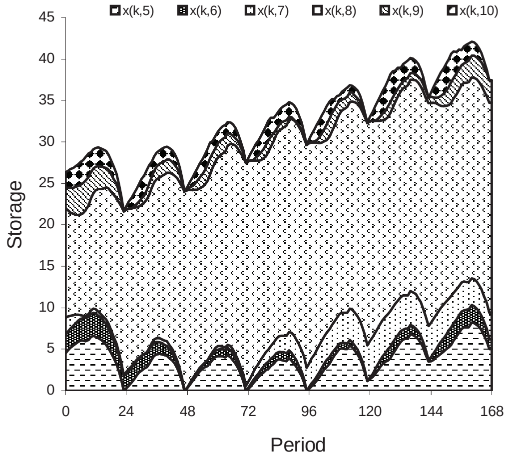
The solution is obtained after a moderate number of iterations with all constraints being satisfied.

The weekly optimal water discharges from each hydroelectric plant, obtained by each strategy for the same conditions, produce a very different and significant storage evolution as well as for the outflow assessment, which are shown in figure 3 and table 3.

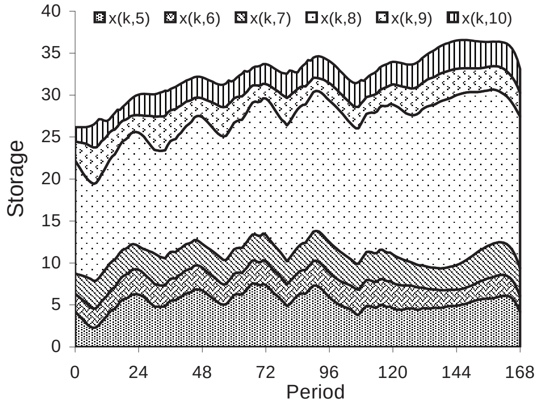
The optimal scheduling of the water discharge conducts to the filling of the upstream reservoirs in comparison with the downstream ones.

We can notice from the figure 3 that the second strategy provides better scheduling results at the end of the planning horizon. Indeed, as it is shown in table 3, the second strategy allows us to have the global storage of the reservoirs at the end of the planning horizon higher than the one obtained with the strategy one.





strategy 1



strategy 2

Fig. 3: Optimal evolution storage trajectories.
(Top: reservoirs 1-4, Bottom: reservoirs 5-10)

Table 3. Initial and final contents of reservoirs

Reservoir number	Initial contents (Mm^3)	Strategy 1 Final contents (Mm^3)	Strategy 2 Final contents (Mm^3)
1	4388,6	4411,73	4411,73
2	493,18	552,72	552,72
3	499,00	396,71	472,5
4	252,5	35,81	42,01
5	4,25	4,16	5,05
6	2,6	1,88	1,13
7	2,4	3,3	3,18
8	13,45	18,12	25,34
9	2,27	2,79	2,70
10	1,7	2,88	0
Total	**5659,65** **100%**	**5430,1** **95,94%**	**5516,36** **97,47%**

By comparing the results of the table 3, we notice that:

- The final contents obtained by using the strategy 2 are greater than ones obtained in the strategy 1.
- The decrease in the total contents in all reservoirs is 4,06% for the strategy 1 and only 2,53% for the strategy 2.
- The amount of water stored in all reservoirs at the end of the planed horizon obtained by the strategy 2 is greater than one found with the strategy 1.

6. Conclusion

In this paper, we have presented a new model for the short-term operating strategy of hydroelectric power systems, which consists to maximize the potential energy of the whole system.
In addition, two strategies for scheduling hydropower systems are presented.

The first strategy is classic, based on the maximization of the reservoir contents at the end of the exploitation horizon. The second strategy is base on the maximazation of the resevoir contents at the end of the sub-periods of the same exploitation horizon.
The second strategy has permitted us to have more potential energy of water stored in all reservoirs at the end of the planed horizon with all operating constraints satisfied.

References

[1] Garcia JM., Turgeon A. Gestion Optimale d'un système Hydro-électrique aspects Long terme et court terme. RAIRO. Automatic systems analysis and control 1981; 15(3): 243-262.

[2] Fosso OB, Belsnes MM. Short-term hydro scheduling in a liberalized power system. POWERCON'2004; 1321-1326.

[3] Sage AP, White CC. Optimum systems control. New Jersey: Prentice-Hall Inc; 1977; p. 126-141, 325-327

[4] Oh YN. An application of the discrete maximum principle to the most economical power-system operation. EE Japan. 1967; 87:17-29.

[5] Pierre DA, Lowe MJ. Mathematical programming via augmented Lagrangian. An introduction with computer programs. Massachusetts. Addison Wesley publishing Co.; 1975, 127-170.

[6] Bertsekas DP. Multiplier methods. Automatica 1976; 12:135-145.

[7] Bertsekas DP. On the method of multipliers for convex programming. IEEE Trans AC 1975; 185-388.

[8] Rodrigues RN, Finardi EC, da Suva EL. Optimal dispatch of hydro generation plants via augmented Lagrangian. IEEE Power Engineering Society General Meeting 2005; 2732–2737.

[9] Ray WH, Soleman MA. The optimal control of processes containing pure time delays. Eleventh joint automatic conference of the America Automatic Control Council 1970; 473-483.

[10] Naresh R, Shatma J. Hydro system scheduling using ANN approach. IEEE Trans PS 2000; 15(1): 388-395

[11] Gary W. Chang, Mohamed A. Experiences with mixed integer linear programming based approaches on short-term hydro scheduling approach. IEEE Trans PS 200; 16(4): 743-749.

[12] Xiaohong G, Svoboda A, Chao-an L. Scheduling hydro power systems with restricted operating zones and discharge ramping constraints. IEEE Trans PS 1999; 14(1): 126-131.

[14] Wang I, Yuan X, Zhang Y. Short-term scheduling of large-scale hydropower systems for energy maximization. International Journal of Power and Energy Systems 2004; 30(3): 198-205.